Decision-Driven Models with Probabilistic Soft Logic

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Introduction

• Personalized medical decisions require integrating ever-increasing amounts of uncertain information, making it more difficult to compute the marginal probability distributions of interest to decision makers.
• We propose a new modeling approach: decision-driven modeling, which reasons probabilistically about marginals.
• We show how decision-driven models can be constructed easily and represented compactly using probabilistic soft logic, a recently introduced framework for statistical relational learning.

Motivating Example

• Joe Black is a patient with test results indicating a chance hasCancer (P, prostate) = 0.4.
• To define a propositional DDM, we must first define the propositions for which we wish to infer marginals, such as A = “Joe has prostate cancer,” B = “Joe’s prostate cancer is aggressive,” and C = “Frank has prostate cancer.”
• Then we define a propositional DDM as follows: Let A = (A1, ..., An) be a set of propositions. A decision-driven model for A is defined by the sample space \( \Omega_p = \{0,1\}^n \), the set of marginal distribution denotations \( M = \{p(x_1), ..., p(x_n)\} \), a probability density function \( f(x = (x_1, ..., x_n)) : \Omega_p \rightarrow \mathbb{R}^n_{+} \) such that \( \int_{x \in [0,1]^n} f(x) dx = 1 \), and a mapping \( g: \Omega_p \rightarrow (M \rightarrow [0,1]) \).

Decision-Driven Models

• Intuitively, a decision-driven model (DDM) is a probability distribution over a set of random variables, each of which represents a probability distribution.
• To define a propositional DDM, we must first define the propositions for which we wish to infer marginals, such as A = “Joe has prostate cancer,” B = “Joe’s prostate cancer is aggressive,” and C = “Frank has prostate cancer.”
• Then we define a propositional DDM as follows: Let A = (A1, ..., An) be a set of propositions. A decision-driven model for A is defined by the sample space \( \Omega_p = \{0,1\}^n \), the set of marginal distribution denotations \( M = \{p(x_1), ..., p(x_n)\} \), a probability density function \( f(x = (x_1, ..., x_n)) : \Omega_p \rightarrow \mathbb{R}^n_{+} \) such that \( \int_{x \in [0,1]^n} f(x) dx = 1 \), and a mapping \( g: \Omega_p \rightarrow (M \rightarrow [0,1]) \).

Probabilistic Soft Logic

• Probabilistic soft logic (PSL) is a language for compactly representing CCMRFs.
• PSL reasons probabilistically about atoms, which are first-order predicates grounded with arguments.
• Each atom corresponds to a continuous random variable in a CCMRF.
• Templates for compatibility functions and constraints are written as rules in first-order logic.
• Example: The compatibility function \( \phi_1 \) can be expressed as the ground rule:

\[
\text{hasCancer}(\text{Frank}, \text{prostate}) \cdot 0.4 \Rightarrow \text{hasCancer}(\text{Joe}, \text{prostate})
\]

An Example Decision-Driven Model

• A natural choice to represent a density function for a propositional DDM is a constrained continuous Markov random field (CCMRF).
• A CCMRF has the following form:

\[
f(x) = \frac{1}{Z(A)} \exp\left(-\sum_{i=1}^{m} \lambda_i \phi_i(x)\right)
\]

along with a set of constraints on \( x \).
• Each \( \phi_i \) is a non-negative compatibility function measuring how compatible dimensions of \( x \) are. The value 0 is perfect compatibility.
• Example: Statistical data show that 2 out of 5 men whose brothers have prostate cancer develop prostate cancer as well. We can represent this as the compatibility function:

\[
\phi_1(\langle P_A = x_1, P_B = x_2, P_C = x_3 \rangle) = \max(0, x_3 \cdot 0.4 - x_1)
\]

Inference

• The MAP state of a DDM is the set of most likely marginal distributions.
• Inference in many DDMs, such as those constructed with PSL, is efficient, since finding the MAP state of the random variables can be formulated as a numerical optimization problem.
• Interpreting the MAP state is straightforward, since users can see how each inferred marginal affected the others.
• Marginal inference in such DDMs can be viewed as computing a confidence measure in the inferred marginals.

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